Math 53: Multivariable Calculus **Sections 102, 103**

Worksheet for 2020-04-17

Conceptual Review

Question 1. Is there a vector field $\mathbf{F}$ on $\mathbb{R}^3$ such that $\nabla \times \mathbf{F} = \langle x, 0, 0\rangle$?

Problems

Problem 1. Let D be the sphere $x^2 + y^2 + z^2 = 9$, oriented positively (outwards).

(a) Without parametrizing, compute the unit normal $\mathbf{n}$ for D in terms of x, y, z. Hint: You did this a lot in Chapter 14.

(b) Compute the flux of the vector field $\mathbf{F} = \langle x, y, z\rangle$ through D. Hint: The surface area of a sphere of radius R is $4\pi R^2$.

Problem 2. The curve $\langle 2 + \cos u, \sin u\rangle$, $0 \le u \le 2\pi$ in the xy-plane is rotated around the y-axis. Parametrize the resulting surface, and describe it.

Problem 3. Compute the flux of the vector field $\langle 0, 0, 4-z^2\rangle$ outwards through the closed cylinder with lateral side $x^2+y^2 = 10$ and lids $z = 0$ and $z = 2$. Note that this surface has three parts.

Problem 4. Compute the divergence of the vector field in Problem 1. Integrate it over the 3d interior of the sphere: $x^2+y^2+z^2 \le 9$. What do you get?

Compute the divergence of the vector field in Problem 3. Integrate it over the 3d interior of the cylinder. What do you get?